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A Critical Estimation of Ideological and Political Education for Sustainable Development Goals Using an Advanced Decision-Making Model Based on Intuitionistic Fuzzy Z-Numbers

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ABSTRACT

Ideological and political education plays a critical role in fostering informed, responsible, and ethically grounded citizens, directly contributing to Sustainable Development Goals such as Quality Education (SDG 4) and Peace, Justice, and Strong Institutions (SDG 16). However, evaluating the educational strategies in this domain often involve complex, uncertain, and subjective judgments. To address this challenge, this study introduces a robust multi-attribute decision-making (MADM) framework leveraging Intuitionistic Fuzzy Z-Numbers (IFZNs), which effectively handle ambiguity and reliability in human evaluations. We propose novel mathematical operations using triperidol intuitionistic fuzzy weighted average operators to aggregate IFZNs under both restriction and reliability components. This framework enhances transparency, inclusivity, and objectivity in policy and curriculum design. A real-world application is demonstrated through a case study that evaluates advanced educational strategies based on diverse attribute information such as accessibility, ethical alignment, critical thinking enhancement, and social impact. Comparative analysis confirms the model's superiority over conventional MADM techniques. This work concludes with insights on implementation, limitations, and directions for future interdisciplinary research, supporting broader agenda of sustainable, inclusive, and values-based education systems.

1. Introduction

Ideological and political education resources represent a strategic asset for cultivating responsible citizenship, civic engagement, and informed decision-making in society. These resources encompass textbooks, governmental policies, digital content, historical narratives, and participatory platforms—all geared toward strengthening societal values, promoting justice, and

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nurturing critical thinking. As recognized in the UN Sustainable Development Goals (SDGs), especially SDG 4 and SDG 16, ensuring equitable access to value-driven education is foundational to peaceful, inclusive, and resilient societies [1]. The primary goal of such education is to cultivate a comprehensive understanding of societal structures, national identity, and civic responsibility. In many countries, these resources aim to promote unity, national pride, and loyalty, with a focus on moral and ethical development. When implemented in educational systems, these resources foster an understanding of governance, rights, and responsibilities, contributing to the holistic development of individuals as responsible and informed citizens [2]. Figure 1 captures the role of Ideological and Political Education in society.

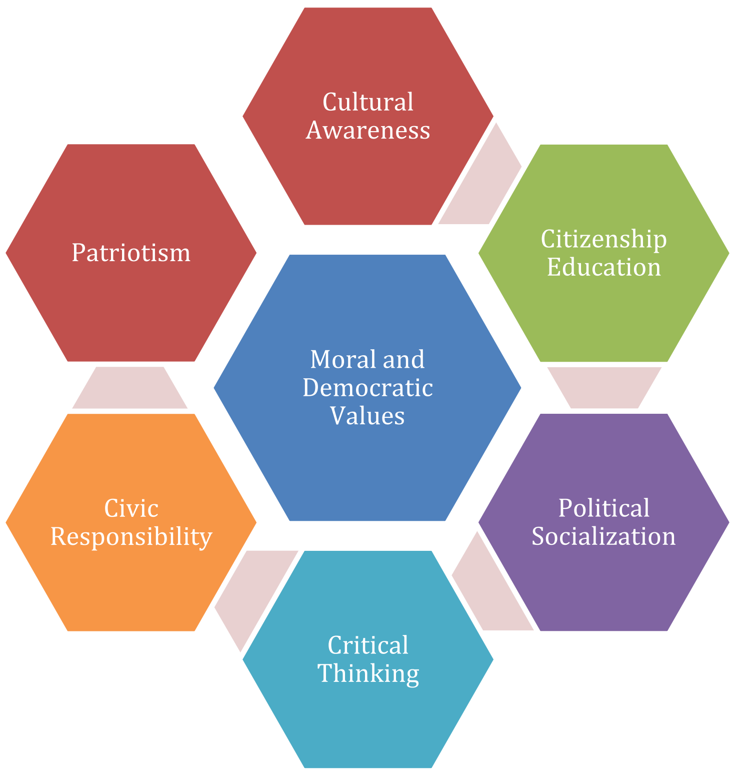

Fig. 1 Role of Ideological and Political Education.

Ideological and political education plays a significant role in shaping talent by instilling values that align with national or organizational goals. By integrating this education into formal curriculums and workplaces, institutions aim to develop individuals who possess technical skills, a deep understanding of societal dynamics, and a commitment to public welfare. Such education reinforces the importance of social responsibility, ethical decision-making, and political awareness in the workforce. In talent management, individuals who have undergone ideological and political education often demonstrate loyalty, discipline, and dedication, aligning with organizational values, particularly in sectors like public administration, education, and defence [3]. This alignment between personal and institutional values helps in retaining and nurturing talent that supports long-term national or organizational goals. Figure 2 elaborates on the different characteristics of the discussed terminology of ideological and political education.

However, ideological and political education can sometimes create challenges in talent management, particularly in dynamic industries that require innovation and adaptability. While promoting ideological conformity can foster unity and consistency, it may also stifle creativity and discourage critical thinking if not implemented with flexibility [4]. Employees or students might feel restricted in expressing diverse viewpoints, leading to a homogeneous thinking pattern that is incompatible with the demands of innovation-driven sectors such as technology, research, and entrepreneurship. Talent management strategies need to balance ideological training with fostering open-mindedness and encouraging individuals to think critically while maintaining ethical standards and societal values [5].

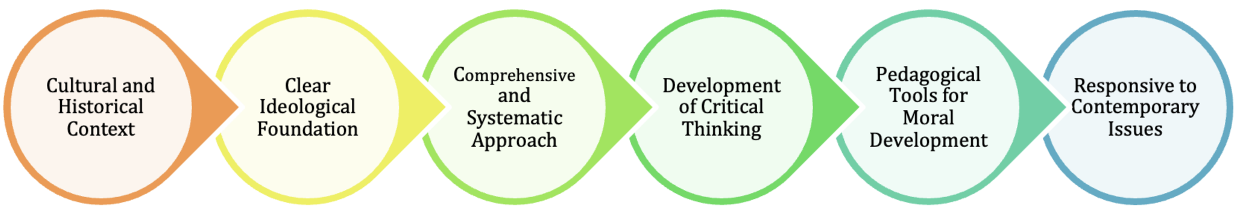

Fig. 2 Characteristics of Ideological and Political Education.

Institutions need a strategic approach to maximize the positive impact of ideological and political education on talent management. Organizations can integrate ideological education into talent development programs by ensuring that such education remains relevant, inclusive, and adaptable [6], [7]. Encouraging dialogue, critical analysis, and open discussion alongside the dissemination of core values allows for a more well-rounded talent pool. Institutions should also focus on aligning ideological education with today's workforce's globalized and multicultural nature. This integration enables individuals to understand national or organizational ideologies while appreciating diverse perspectives, creating well-rounded talent capable of contributing to local and international goals [8].

The theory of fuzzy sets (FSs) was introduced by Zadeh [9] in 1965 to address situations involving vagueness and uncertainty in information, which traditional binary logic could not adequately handle. Classical set theory operates on a binary notion, where elements either belong to a set or do not, making it difficult to model real-world situations that often contain ambiguity. The FSs allows for partial membership, where an element can belong to a set to a certain degree ranging from 0 to 1. This flexibility makes fuzzy sets highly useful in areas like control systems, decision-making, pattern recognition, and artificial intelligence, where decisions must often be made with imprecise or incomplete data.

Furthermore, Zadeh [10] expanded the concepts of linguistic variables for different real-life applications pattern recognition, artificial intelligence, social and economic sectors, and medical diagnosis. Zadeh [11] applied the theory of fuzzy logic to resolving different complicated issues related to neural networks and Machine Intelligence Quotient problems. Later on, Atanassov [12] gave an innovative theory of an intuitionistic fuzzy set (IFS) by exploring the concepts of FSs into two different components such as positive membership grade (PMG) and negative membership grade (NMG), which are bounded by closed interval $[0,1]$. Atanassov [13], [14] also expanded the theory of IFSs to derive some fundamental rules and properties. A number of applications and experimental case studies are resolved based on the concept of FSs and IFSs seen in [15], [16], [17].

In our methodology, Z-numbers which were created by Zadeh [18] handle reliability. The Z-number consists of two parts. A restriction function is the first component, and a reliability function for the first component is the second. Z-numbers are utilized in calculations involving fuzzy numbers that are not always accurate. Numerous MCDM techniques are employed in literature with Z-numbers. Furthermore, Sari and Kahraman [19] combined theories of two different concepts intuitionistic fuzzy numbers and Z-numbers to derive a novel concept of intuitionistic Z-numbers (IZNMs). There are many applications of IZNMs that exist in the literature. The strength of IZNMs is to facilitate more reliable and authentic results by resolving uncertain information of experts' opinions.

In decision-making processes, aggregation operators play a crucial role in synthesizing diverse information from multiple sources into a single value, enabling more informed and balanced decisions. These operators combine individual preferences, opinions, or criteria evaluations, considering their relative importance or uncertainty. Many mathematicians derived various fuzzy terminologies and aggregation operators. For instance, Hussain *et al.*, [20] investigated Dombi Hamy mean models to derive new mathematical approaches to t-spherical fuzzy information. Hussain *et al.*, [21] proposed Sugeno-Weber aggregation operators to assess advanced digital security techniques based on the MADM problem. Hussain *et al.*, [22] constructed Aczel Alsina aggregation operators to deal with uncertain human opinions and complicated real-life applications. Xiong *et al.*, [23] evaluated expert's opinions related to the agriculture sector using Hamy mean aggregation operators. Wang *et al.*, [24] proposed mathematical approaches of Sugeno-Weber aggregation operators to investigate unknown degree of weights and selection of solar panels. Hussain *et al.*, [25] evaluated different construction materials using intuitionistic fuzzy framework and decision-making models. Riaz and Farid [26] discussed some reliable characteristics of supply chain enterprises based on the decision analysis process and Linear Diophantine fuzzy discipline. Riaz *et al.*, [27] combined two different theories of distance and entropy measures to propose mathematical approaches to Einstein aggregation operators. Riaz *et al.*, [28] discussed a novel theory of decision-making methods for resolving the judgments of a group of expert's. Akram and Bilal [29] investigated the Homotopy perturbation method taking into account bipolar fuzzy information. Akram *et al.*, [30] extended the theory of the decision-making problem of the MARCOS method based on 2-tuple linguistic q-rung orthopair picture fuzzy environments. Senapati [31] developed Aczel Alsina aggregation operators by incorporating the theory of a single-valued neutrosophic framework. Hussain *et al.*, [32] discussed ranking methods for preferences using complex spherical fuzzy systems and Aczel Alsina aggregation operators. Senapati *et al.*, [33] anticipated some new geometric aggregation operators based on an intuitionistic fuzzy system and Aczel Alsina t-norms. Hussain *et al.*, [34] evaluated some reasonable transportation using Heronian mean aggregation models and T-spherical fuzzy Aczel Alsina t-norms. Hussain *et al.*, [35] examined profiles of different research scholars using a decision analysis process with Azel Alsina aggregation operators. Senapati [36] designed robust mathematical approaches for handling uncertain and ambiguous human information using a decision analysis process. Ali *et al.*, [37] used the theory of Bonferroni mean models to derive new aggregation operators and TOPSIS method for evaluating suitable optimal options. Garg [38] discussed mathematical approaches of picture fuzzy information. Garg [39] established some novel approaches of trigonometric functions and q-rung orthopair fuzzy system.

Abiyev *et al.*, [40] designed fuzzy-controlled algorithms based on the theory of Z-numbers. Ashraf *et al.*, [41] deduced Sugeno-Weber aggregation operators to assess the impact of climate change due to greenhouse gas emissions. Ghoushchi and Sarvi [42] applied concepts of Z-numbers using the decision-making approach of the SWARA and MOORA methods. Ye [43] generalized similarity measures to design new aggregation operators based on neutrosophic Z-numbers. Haktanır and Kahraman [44] selected the most favorable hydrogen storage technologies using the TOPSIS method and the dominant approach of Z-numbers. Liu *et al.*, [45] introduced a novel concept of Linguistic q-Rung Orthopair fuzzy Z-number for handling uncertain human information and selecting a suitable airline aircraft based on decision-making problems. Wang and Mao [46] investigated unknown weights of criteria and suitable optimal options using the decision-making technique of the TOPSIS method.

MOTIVATION BEHIND PROPOSED RESEARCH WORK

Intuitionistic fuzzy z-numbers combine the concepts of intuitionistic fuzzy sets (IFS) and z-numbers, aiming to handle decision-making problems that involve uncertainty and imprecision more effectively. The motivation behind using intuitionistic fuzzy z-numbers is to provide a richer framework for representing information that has both degrees of membership and non-membership (from IFS) along with reliability assessments of the information (from z-numbers). This dual nature allows decision-makers to account not only for how true or false a piece of information might be but also for how reliable this judgment is. This is particularly useful in real-world situations where both uncertainty and hesitation are inherent, such as in risk analysis, financial decisions, or situations requiring expert judgments.

The motivation behind integrating ideological and political education resources into talent management is to align the workforce with the ethical, political, and cultural values of an organization or nation. This approach is used to ensure that talent is not only skilled but also ideologically aligned with the goals and aspirations of the entity they work for. In national contexts, it fosters loyalty, civic responsibility, and national identity, which are crucial for stability and social cohesion. In organizations, it strengthens corporate culture, helping individuals internalize the organization's mission and values. This alignment can contribute to higher employee retention, motivation, and a cohesive work environment where the workforce's beliefs and behaviors are consistent with institutional goals.

The MADM problems arise when decisions must be made based on multiple conflicting criteria. The motivation behind developing frameworks for solving MADM problems is the need to assist decision-makers in systematically evaluating and selecting the best option among many alternatives, especially when each option has both strengths and weaknesses across various attributes. Such problems are common in business, engineering, public policy, and everyday life, where trade-offs between attributes like cost, quality, efficiency, and risk need to be considered. The development of MADM methodologies is motivated by the desire to reduce subjective biases in decision-making, improve the rationality of the selection process, and ensure that all relevant factors are appropriately weighted and considered in complex decision scenarios. The main contributions of this presentation are explored as follows:

a) To articulate the theory of intuitionistic fuzzy Z-numbers for handling uncertain and vague type information of human opinions.
b) We discussed some feasible operations of IZNs for aggregating uncertain and vague-type human judgments.
c) An intelligent decision-making model of the MADM problem is established to achieve the ranking of alternatives based on certain criteria.
d) Additionally, an application related to ideological and political education resources is articulated to evaluate advanced educational strategies under some prominent attribute information.
e) We constructed a comprehensive comparative study to validate the reliability of diagnosed methodologies and aggregation operators.

The remaining part of this presentation is articulated as follows: Section 2 presents a basic overview of FSs, IFSs, and Z-numbers. Section 3 established an intelligent decision algorithm for the MADM problem under consideration of the theory of intuitionistic fuzzy Z-numbers. In section 4, we discussed an experimental case about ideological and political education with the help of proposed methodologies. A comprehensive comparison method is established to prove the validation and supremacy of derived theories in section 5. Finally, we summarized the whole article with remarkable comments in section 6.

1. Basic Concepts

This section facilitates a basic overview of FSs and IFSs to propose new methodologies of an intuitionistic fuzzy Z-number.

Definition 1: [9] Consider $\mathcal{D}$ be a universal set and a FS $\mathcal{H}$ is defined as follows:

$$\mathcal{H} = \{x, u(x) | x \in \mathcal{D}\}$$

Moreover, $u(x) \in [0,1]$ denotes a positive membership value of $\mathcal{H}$ in $\mathcal{D}$.

Definition 2: [12] An IFS $\mathcal{K}$ is defined as follows:

$$\mathcal{K} = \{x, (u(x), v(x)) | x \in \mathcal{D}\}$$

It's clear that $u(x) \in [0,1]$ and $v(x) \in [0,1]$ represent positive membership value and negative membership value of $\mathcal{K}$ in $\mathcal{D}$ with subject to the condition:

$$0 \leq u(x) + v(x) \leq 1$$

Furthermore, the hesitancy value of an IFS is denoted by $\ddot{\upsilon}(x) = 1 - (u(x) + v(x))$. An intuitionistic fuzzy value (IFV) is denoted by $\mathcal{Y} = (u(x) + v(x))$.

Figure 3 demonstrates trapezoidal IFV $\mathcal{L} = \{(\breve{\Upsilon}_2, \breve{\Upsilon}_3, \breve{\Upsilon}_4, \breve{\Upsilon}_5)(\breve{\Upsilon}_1, \breve{\Upsilon}_3, \breve{\Upsilon}_4, \breve{\Upsilon}_5)\}$. The trapezoidal IFV is used to fuse uncertain human information instead of triangular IFV.

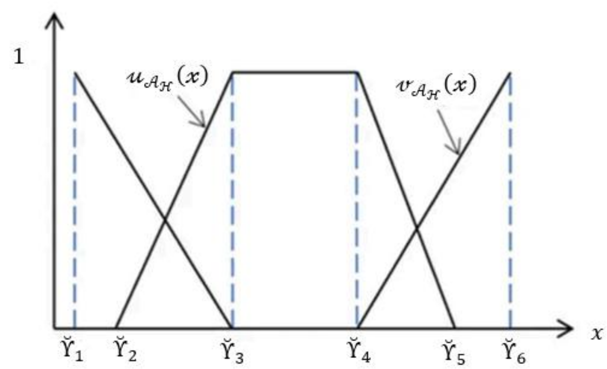

Fig 3. Explore the shape of trapezoidal intuitionistic fuzzy numbers.

Z-Numbers

The theory of Z-numbers proposed by Zadeh [18] covers two components the restriction $\mathcal{E}$ and reliability $\mathcal{F}$components. The concepts of Z-numbers include probability concepts in the fuzzy domains, as the reliability $\mathcal{F}$ component deals with the statistical discipline of probability distribution. In Figure 4, restriction $\mathcal{E}$ and reliability $\mathcal{F}$components represented by trapezoidal fuzzy value and triangular fuzzy value respectively. For more understanding of Z-numbers, a geometrical representation of Figure 4 also illustrates the behavior of the Z-numbers.

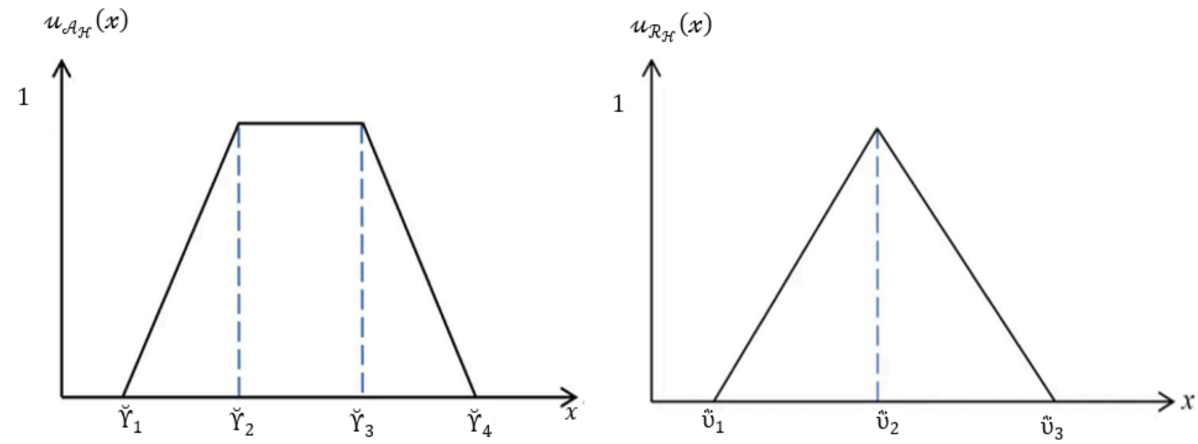

Fig 4. Demonstrate the graphical shape of Z-numbers.

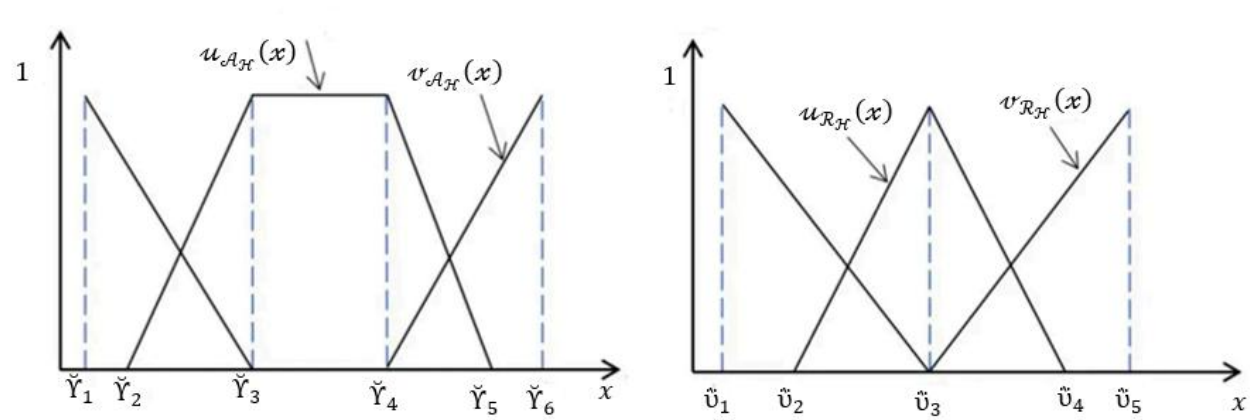

Fig 5. Explore the shape of intuitionistic fuzzy Z-numbers.

2. Intuitionistic Z-Number

Here, the Z-number can be written as positive membership value and negative membership value, Sari and Kahraman [19] derived concepts of intuitionistic Z-numbers.

Definition 3: [19] Consider $\mathcal{D}$ be a universal set and an intuitionistic Z-number $\mathcal{H} = (\mathcal{A}_{\mathcal{H}}, \mathcal{R}_{\mathcal{H}})$ is given by:

$$\mathcal{A}_{\mathcal{H}} = \left\{x, \left(u_{\mathcal{A}_{\mathcal{H}}}(x), v_{\mathcal{A}_{\mathcal{H}}}(x)\right) | x \in \mathcal{D}\right\} = \{\check{Y}_2, \check{Y}_3, \check{Y}_4, \check{Y}_5; \check{Y}_1, \check{Y}_3, \check{Y}_4, \check{Y}_5\}$$

$$\mathcal{R}_{\mathcal{H}} = \left\{x, \left(u_{\mathcal{R}_{\mathcal{H}}}(x), v_{\mathcal{R}_{\mathcal{H}}}(x)\right) | x \in \mathcal{D}\right\} = \{\check{\ddot{\upsilon}}_2, \check{\ddot{\upsilon}}_3, \check{\ddot{\upsilon}}_4, \check{\ddot{\upsilon}}_5; \check{\ddot{\upsilon}}_1, \check{\ddot{\upsilon}}_3, \check{\ddot{\upsilon}}_4, \check{\ddot{\upsilon}}_5\}$$

Additionally, intuitionistic Z-numbers are illustrated in Figure 5 as follows:

Let $\mathcal{A}_I = \{x, u_{\mathcal{A}}(x), v_{\mathcal{A}}(x) | x \in E\} = (\breve{Ÿ}_2, \breve{Ÿ}_3, \breve{Ÿ}_4, \breve{Ÿ}_5; \breve{Ÿ}_1, \breve{Ÿ}_3, \breve{Ÿ}_4, \breve{Ÿ}_6)$ and $\mathcal{R}_I = \{x, u_{\mathcal{R}}(x), v_{\mathcal{R}}(x) | x \in E\} = (\ddot{\text{ʊ}}_2, \ddot{\text{ʊ}}_3, \ddot{\text{ʊ}}_4; \ddot{\text{ʊ}}_1, \ddot{\text{ʊ}}_3, \ddot{\text{ʊ}}_5)$ where $u_{\mathcal{A}}(x)$ is a trapezoidal belonging function and $u_{\mathcal{R}}(x)$ is a triangular un-belonging function.

The Intuitionistic Z-number is given by:

$$u_{\mathcal{A}_I}(x) = \begin{Bmatrix} \frac{x - \breve{Ÿ}}{\breve{Ÿ}_3 - \breve{Ÿ}_2}, if\ \breve{Ÿ}_2 \le x \le \breve{Ÿ}_3 \\ \frac{\breve{Ÿ}_4 - x}{\breve{Ÿ}_4 - \breve{Ÿ}_3}, if\ \breve{Ÿ}_3 \le x \le \breve{Ÿ}_4 \\ 0 \quad otherwise \end{Bmatrix}$$

$$v_{\mathcal{A}_I}(x) = \begin{Bmatrix} \frac{\breve{Ÿ}_3 - x}{\breve{Ÿ}_3 - \breve{Ÿ}_1}, if\ \breve{Ÿ}_1 \le x \le \breve{Ÿ}_3 \\ \frac{x - \breve{Ÿ}_3}{\breve{Ÿ}_5 - \breve{Ÿ}_3}, if\ \breve{Ÿ}_3 \le x \le \breve{Ÿ}_5 \\ 0 \quad otherwise \end{Bmatrix}$$

$$u_{\mathcal{R}_I}(x) = \begin{Bmatrix} \frac{x - \ddot{\text{ʊ}}_2}{\ddot{\text{ʊ}}_3 - \ddot{\text{ʊ}}_2}, if\ \ddot{\text{ʊ}}_2 \le x \le \ddot{\text{ʊ}}_3 \\ \frac{\ddot{\text{ʊ}}_4 - x}{\ddot{\text{ʊ}}_4 - \ddot{\text{ʊ}}_3}, if\ \ddot{\text{ʊ}}_3 \le x \le \ddot{\text{ʊ}}_4 \\ 0 \quad otherwise \end{Bmatrix}$$

$$v_{\mathcal{R}_I}(x) = \begin{Bmatrix} \frac{\ddot{\text{ʊ}}_3 - x}{\ddot{\text{ʊ}}_3 - \ddot{\text{ʊ}}_1}, if\ \ddot{\text{ʊ}}_1 \le x \le \ddot{\text{ʊ}}_3 \\ \frac{x - \ddot{\text{ʊ}}_3}{\ddot{\text{ʊ}}_5 - \ddot{\text{ʊ}}_3}, if\ \ddot{\text{ʊ}}_3 \le x \le \ddot{\text{ʊ}}_5 \\ 0 \quad otherwise \end{Bmatrix}$$

The procedure of defuzzification of IF Z-number is given by"

Step 1: Transform the reliability component into crisp values:

$$\mathcal{O}_I = \frac{\ddot{\text{ʊ}}_2 + 2\ddot{\text{ʊ}}_3 + \ddot{\text{ʊ}}_4}{4} + \frac{\ddot{\text{ʊ}}_1 + 2\ddot{\text{ʊ}}_3 + \ddot{\text{ʊ}}_5}{\mathcal{G}} \tag{1}$$

Where $\mathcal{G}$ be any large non-negative numbers.

Step 2: Investigate the weighted Z-number of restriction components using the crisp value of the reliability component.

$$Z^{\mathcal{O}_I} = \left\{ \left(x, u_{\mathcal{A}_{\mathcal{H}}^{\mathcal{O}}}(x) \right) | u_{\mathcal{A}_{\mathcal{H}}^{\mathcal{O}_I}}(x) = \mathcal{O}_I u_{\mathcal{A}_{\mathcal{H}}}(x), u(x) \in [0,1] \right\} \tag{2}$$

Step 3: Convert $Z^{\mathcal{O}_I}$ into ordinary fuzzy Z-number:

$$\hat{\mathcal{Z}} = \left\{\left(x, u_{\hat{\mathcal{Z}}}(x)\right) | u_{\hat{\mathcal{Z}}}(x) = u_{\mathcal{A}_{\mathcal{H}}}\left(\frac{x}{\sqrt{\mathcal{O}_I}}\right), u(x) \in [0,1]\right\} \tag{3}$$

Let $\mathcal{A}_{I\delta} = \{x, u(x), v(x); \delta | x \in E\} = \left(\check{\Upsilon}_2, \check{\Upsilon}_4, \check{\Upsilon}_5, \check{\Upsilon}_7, \delta_1\right) = \left(\check{\Upsilon}_1, \check{\Upsilon}_3, \check{\Upsilon}_6, \check{\Upsilon}_8, \eta_1\right)$ $\mathcal{R}_{I\beta} = \{x, u_{\mathcal{R}}(x), v_{\mathcal{R}}(x); \beta | x \in E\} = (\ddot{\upsilon}_2, \ddot{\upsilon}_3, \ddot{\upsilon}_4; \delta_2), (\ddot{\upsilon}_1, \ddot{\upsilon}_3, \ddot{\upsilon}_5; \eta_2)$ where $u_{\mathcal{A}}^{\delta}(x)$ is trapezoidal belonging function; $u_{\mathcal{R}}^{\beta}(x)$ is a triangular un-belonging function; δ is the height of the triangular or trapezoidal belonging function; and η is the lowest value of the triangular or trapezoidal un-belonging function. The functions are defined by Eqs. 4-8.

$$u_{\mathcal{A}_I}^{\delta_1}(x) = \begin{cases} \dfrac{x - \check{\Upsilon}_2}{\check{\Upsilon}_4 - \check{\Upsilon}_2}\delta_1, if\ \check{\Upsilon}_2 \le x \le \check{\Upsilon}_4\ \delta_1, if \check{\Upsilon}_4 \le x \le \check{\Upsilon}_5 \\ \dfrac{\check{\Upsilon}_7 - x}{\check{\Upsilon}_7 - \check{\Upsilon}_5}\delta_1, if\ \check{\Upsilon}_5 \le x \le \check{\Upsilon}_7 \\ 0 \quad otherwise \end{cases} \tag{4}$$

$$v_{\mathcal{A}_I}^{\eta_1}(x) = \begin{cases} \left(\dfrac{\eta_1 - x}{\check{\Upsilon}_3 - \check{\Upsilon}_1}\right)x + \left(\dfrac{\check{\Upsilon}_3 - \eta\check{\Upsilon}_1}{\check{\Upsilon}_3 - \check{\Upsilon}_1}\right), if\ \check{\Upsilon}_1 \le x \le \check{\Upsilon}_3\ \eta_1, if\ \check{\Upsilon}_3 \le x \le \check{\Upsilon}_6 \\ \left(\dfrac{1 - \eta_1}{\check{\Upsilon}_8 - \check{\Upsilon}_6}\right)x + \left(\dfrac{\eta_1\check{\Upsilon}_8 - \check{\Upsilon}_6}{\check{\Upsilon}_8 - \check{\Upsilon}_6}\right), if\ \check{\Upsilon}_6 \le x \le \check{\Upsilon}_8 \\ 0 \quad otherwise \end{cases} \tag{5}$$

$$u_{\mathcal{R}_I}^{\delta_2}(x) = \begin{cases} \dfrac{x - \ddot{\upsilon}_2}{\ddot{\upsilon}_3 - \ddot{\upsilon}_2}\delta_2, if\ \ddot{\upsilon}_2 \le x \le \ddot{\upsilon}_3 \\ \dfrac{\ddot{\upsilon}_4 - x}{\ddot{\upsilon}_4 - \ddot{\upsilon}_3}\delta_2, if\ \ddot{\upsilon}_3 \le x \le \ddot{\upsilon}_4 \\ 0 \quad otherwise \end{cases} \tag{7}$$

$$v_{\mathcal{R}_I}^{\eta_2}(x) = \begin{cases} \left(\dfrac{\eta_2 - x}{\ddot{\upsilon}_3 - \ddot{\upsilon}_1}\right)x + \left(\dfrac{\ddot{\upsilon}_3 - \eta_2\ddot{\upsilon}_1}{\ddot{\upsilon}_3 - \ddot{\upsilon}_1}\right), if\ \ddot{\upsilon}_1 \le x \le \ddot{\upsilon}_3 \\ \left(\dfrac{1 - \eta_2}{\ddot{\upsilon}_5 - \ddot{\upsilon}_3}\right)x + \left(\dfrac{\eta_2\ddot{\upsilon}_5 - \ddot{\upsilon}_3}{\ddot{\upsilon}_5 - \ddot{\upsilon}_3}\right), if\ \ddot{\upsilon}_3 \le x \le \ddot{\upsilon}_5 \\ 0 \quad otherwise \end{cases} \tag{8}$$

The reliability function is converted to a classical number by Eq 9.

$$\alpha_I = \left(\frac{\ddot{\upsilon}_2 + 2\ddot{\upsilon}_3 + \ddot{\upsilon}_4}{4}\right)\delta_2 + \left(\frac{\ddot{\upsilon}_1 + 2\ddot{\upsilon}_3 + \ddot{\upsilon}_5}{\tau}\right)(1 - \eta_2) \tag{9}$$

Then weighted $Z_{\delta_1,\delta_2}^{\alpha}$ number can be obtained by Eq 10.

$$Z_{\delta_1,\eta_1} = \left\{x, u_{\mathcal{A}_I^{\alpha_I}}^{\delta_1}(x), v_{\mathcal{A}_I^{\alpha_I}}^{\eta_1}(x) = u_{\mathcal{A}_I^{\alpha_I}}^{\delta_1}(x) = \alpha_I u_{\mathcal{A}_I^{\alpha_I}}^{\delta_1}(x), v_{\mathcal{A}_I^{\alpha_I}}^{\eta_1}(x) = \alpha_I v_{\mathcal{A}_I^{\alpha_I}}^{\eta_1}, u(x), v(x) \in [0,1]\right\} \tag{10}$$

The weighted $Z_{\delta_1,\delta_2}^{\alpha}$ number is converted to a type 1 fuzzy number by Eq. 11.

$$Z_{\delta_1,\eta_1} = \left\{ \begin{array}{c} x, u_{Z_I}^{\delta_1}(x), v_{Z_I}^{\eta_1}(x) u_{\mathcal{A}}^{\delta_1}(x), \\ = u_{\mathcal{A}}^{\delta_1}\left(x\left(\frac{u_Z^{\delta_1}(x)}{\left(\frac{\ddot{\upsilon}_2 + 2\ddot{\upsilon}_3 + \ddot{\upsilon}_4}{4}\right)\delta_2 + \left(\frac{\ddot{\upsilon}_1 + 2\ddot{\upsilon}_3 + \ddot{\upsilon}_5}{\tau}\right)(1-\eta_2)}, u(x), v(x) \in [0,1] \right)\right), \\ v_{\mathcal{A}}^{\eta_1}\left(x\left(\frac{v_Z^{\eta_1}(x)}{\left(\frac{\ddot{\upsilon}_2 + 2\ddot{\upsilon}_3 + \ddot{\upsilon}_4}{4}\right)\delta_2 + \left(\frac{\ddot{\upsilon}_1 + 2\ddot{\upsilon}_3 + \ddot{\upsilon}_5}{\tau}\right)(1-\eta_2)} \right)\right) \end{array} \right\} \quad (11)$$

3. MADM for Intuitionistic Z-Numbers

The MADM problem is established for handling uncertain information of human opinions in the form of IZNs. To aggregate such information, we applied a trapezoidal intuitionistic fuzzy weighted average (TIFWA) operator. We also used ranking functions of intuitionistic value under a system of centroid to integrate the rank of alternatives or individuals. An algorithm for the MADM problem is designed as follows:

Table 1. Linguistic Scales of IZNs for Restriction Component.

Linguistic scales	Trapezoidal IZNs
Very important (VI)	$\{(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)\}$
Important (I)	$\{(0.75, 0.85, 0.95, 1.0), (0.75, 0.85, 0.95, 1.0)\}$
Medium important (MI)	$\{(0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95)\}$
Medium (M)	$\{(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)\}$
Medium bad (MB)	$\{(0.15, 0.25, 0.35, 0.45), (0.05, 0.25, 0.35, 0.55)\}$
Bad (B)	$\{(0.0, 0.15, 0.25, 0.35), (0.0, 0.15, 0.25, 0.35)\}$
Very bad (VB)	$\{(0.0, 0.0, 0.0, 0.0), (0.0, 0.0, 0.0, 0.0)\}$

Table 2 Linguistic Scales of IZNs for Reliability Component.

Linguistic scales	Trapezoidal IZNs
Not confirm (NC)	$\{(0.05, 0.25, 0.45), (0.05, 0.25, 0.45)\}$
Not very confirm (NVC)	$\{(0.25, 0.45, 0.65), (0.15, 0.45, 0.75)\}$
Confirm (C)	$\{(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)\}$
Very Confirm (VC)	$\{(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)\}$

Step 1: Expert judgments are obtained and converted into IZNs. Table 1 presents linguistic information for two different components such as restriction and reliability terms.

Step 2: Transform IZNs into IFVs using Eqs. 1-3.

Step 3: Applied TIFWA operator to aggregate expert's information.

$$TIFWA(A_1, A_2, A_3, \dots, A_n) = \left(\begin{array}{c} \left(\sum_{i=1}^{n} w_i \mathcal{E}_{i_2}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_3}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_4}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_5} \right), \\ \left(\sum_{i=1}^{n} w_i \mathcal{E}_{i_1}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_3}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_4}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_6} \right) \end{array} \right)$$

Step 4: Aggregate information of all criteria using the TIFWA operator.

Step 5: Ranking of alternatives based on centroid as follows:

$$\mathcal{M}(I_A) = \sqrt{\frac{1}{2}([x_u(I_A) + y_u(I_A)]^2 + [x_v(I_A) + y_v(I_A)]^2)} \tag{9}$$

$$x_u(I_A) = \frac{1}{3}\left(\frac{\check{Y}_4^2 + \check{Y}_5^2 - \check{Y}_2^2 - \check{Y}_3^2 - \check{Y}_2\check{Y}_3 + \check{Y}_4\check{Y}_5}{\check{Y}_4 + \check{Y}_5 - \check{Y}_2 - \check{Y}_3}\right)$$

$$y_u(I_A) = \frac{1}{3}\left(\frac{2\check{Y}_6^2 - 2\check{Y}_1^2 + 2\check{Y}_3^2 + 2\check{Y}_4^2 + \check{Y}_1\check{Y}_3 - \check{Y}_4\check{Y}_6}{\check{Y}_4 + \check{Y}_6 - \check{Y}_1 - \check{Y}_3}\right)$$

$$x_v(I_A) = \frac{1}{3}\left(\frac{\check{Y}_2 + 2\check{Y}_3 - 2\check{Y}_4 - \check{Y}_5}{\check{Y}_2 + \check{Y}_3 - \check{Y}_4 - \check{Y}_5}\right)$$

$$y_v(I_A) = \frac{1}{3}\left(\frac{2\check{Y}_1 + 2\check{Y}_3 - \check{Y}_4 - 2\check{Y}_6}{\check{Y}_1 + \check{Y}_3 - \check{Y}_4 - \check{Y}_6}\right)$$

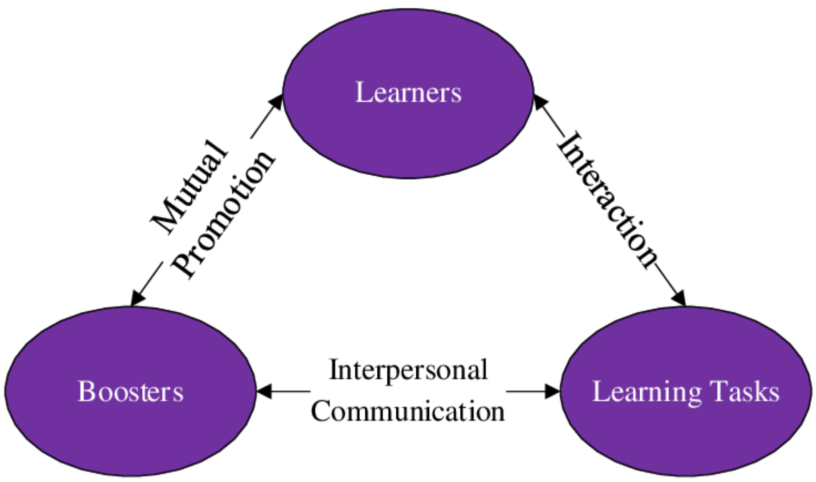

Fig 6. Communication process for Political Education.

4. Case Study

The study of basic problems in ideological and political education (IPE) is vital because it lays the foundation for cultivating individuals with a solid understanding of national identity, values, and political consciousness. IPE, particularly in educational systems like China's, aims to foster loyalty to the state, promote ethical behavior, and ensure that individuals align with the ideological goals of the nation. By addressing basic issues, such as understanding core principles, political theories, and methods of teaching, IPE plays a significant role in shaping the worldview of students and future citizens. This education creates a sense of shared purpose, promoting unity and national development.

Additionally, exploring the basic problems of IPE allows educators and policymakers to address challenges like the gap between ideological theory and practice, the effectiveness of pedagogy, and the adaptation of traditional ideologies to modern, globalized contexts. It helps in identifying the most effective ways to teach political and ideological content in an engaging, relevant manner, ensuring that students can critically understand and apply these concepts. As societies evolve, the constant study and revision of IPE ensures that it remains pertinent, fostering informed and responsible citizens who contribute to societal progress.

In the context of talent management, ideological and political education resources also provide a framework for developing critical thinking and problem-solving abilities in Figure 6. By engaging with complex social, political, and ethical issues, individuals learn to navigate challenges, make informed decisions, and develop innovative solutions that consider both individual and collective well-being. This holistic development process ensures that talent is not only technically proficient but also socially responsible, adaptable, and capable of contributing to sustainable development. In turn, this strengthens the talent pool, improves retention, and supports the strategic objectives of organizations or institutions aiming to cultivate well-rounded, competent leaders and professionals. In this experimental case study, we discuss some appropriate methodologies for the improvement of ideological and political education as follows:

Curriculum-Based Learning $Ꞡ_1$: This approach involves the development of structured educational programs that incorporate ideological and political theories, principles, and case studies. Organizations design courses or modules that align with their core values and objectives, ensuring that employees gain a foundational understanding of the political and ideological contexts relevant to their roles. This method often includes assessments to measure comprehension and application.

Experiential Learning $Ꞡ_2$: Experiential learning emphasizes practical engagement with ideological and political concepts through real-life experiences. This can involve simulations, role-playing exercises, or community service projects that require employees to apply their knowledge in authentic situations. By engaging directly with the material, employees can develop a deeper understanding of the implications and applications of ideological and political education in their work.

Mentorship and Peer Learning $Ꞡ_3$: Leveraging mentorship programs and peer learning opportunities can effectively integrate ideological and political education into talent management. Experienced leaders or colleagues can guide less experienced employees through discussions, workshops, or one-on-one coaching sessions. This informal approach fosters a collaborative learning environment where individuals can share insights, experiences, and perspectives on ideological and political issues.

Online and Blended Learning $\mathbb{A}_4$: E-learning platforms and blended learning models combine traditional face-to-face education with online resources to enhance ideological and political education. This methodology provides flexibility and accessibility while incorporating diverse learning modalities, such as videos, webinars, and interactive forums. Online learning can also facilitate access to a wider range of expert opinions and resources, enriching the educational experience.

Critical Reflection and Discussion $\mathbb{A}_5$: Encouraging critical reflection and open discussions about ideological and political topics fosters a culture of inquiry and debate within organizations. This methodology often involves facilitated group discussions, workshops, or seminars where employees can explore differing viewpoints, analyze current events, and consider the implications of ideological and political beliefs in their professional and personal lives. Promoting an environment where diverse perspectives are welcomed can enhance critical thinking and ethical decision-making among employees.

The above-discussed methodologies are evaluated under the following key features:

Flexibility and Accessibility $\mathfrak{J}_1$**:** E-learning platforms allow employees to access educational materials anytime and anywhere, accommodating different schedules and learning preferences. This flexibility ensures that all employees, regardless of their location or work commitments, can engage with the content.

Scalability $\mathfrak{J}_2$**:** Organizations can easily scale e-learning programs to accommodate large numbers of employees across various locations. This is particularly beneficial for multinational companies that need to provide consistent ideological and political education to diverse teams.

Interactive Learning $\mathfrak{J}_3$**:** E-learning platforms often incorporate interactive elements like quizzes, simulations, and discussion forums, enhancing engagement and participation. This interactivity fosters deeper understanding and encourages critical thinking among learners.

Data Analytics and Tracking $\mathfrak{J}_4$**:** E-learning platforms provide valuable data on learner engagement and progress, enabling organizations to assess the effectiveness of their training programs. This information can inform future training initiatives and help identify areas for improvement.

Personalized Learning Experiences $\mathfrak{J}_5$**:** E-learning platforms can offer personalized learning paths based on individual needs, preferences, and skill levels. This customization ensures that employees receive targeted content that aligns with their roles and career aspirations, enhancing their overall development in ideological and political education.

A. PROCEDURE OF THE MADM PROBLEM UNDER THE SYSTEM OF IZNs

This section demonstrates the stepwise decision algorithm of the MADM problem for evaluating suitable optimal options under consideration of various characteristics and features.

Step 1: The decision-maker arranges their judgments using linguistic scales of restriction and reliability terms of Table 1 and 2 respectively. So, Table 3 demonstrates expert opinions in the form of different criteria against each alternative. Table 4 covers judgments of expert opinions in the form of IZNs.

Step 2: In this step, convert IZNs into IFVs based on Eqs-5-7 considering the value of $\mathcal{G} = 100$. The MGs obtained by defuzzied the value of reliability components of IZNs. The presence of the smaller defuzzied value for the NMGs exhibits a degree of hesitancy. If $\mathcal{G}$ is too big, then the NMGs of the reliability component will be completely ignored. Hence, choosing $\mathcal{G} = 100$ is perfect enough for the purpose of de-fuzzifying IZNS into IFVs.

Step 3: Using the TIFWA operator, aggregate human opinions associated with each criterion and aggregated results are shown in Table 5.

$$TIFWA(A_1, A_2, A_3, \dots, A_n) = \left(\begin{array}{c} \left(\sum_{i=1}^{n} w_i \mathcal{E}_{i_2}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_3}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_4}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_5} \right), \\ \left(\sum_{i=1}^{n} w_i \mathcal{E}_{i_1}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_3}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_4}, \sum_{i=1}^{n} w_i \mathcal{E}_{i_6} \right) \end{array} \right)$$

Table 3. Expert Judgements in the Form of IZNs.

	$\mathfrak{J}_1$	$\mathfrak{J}_2$	$\mathfrak{J}_3$	$\mathfrak{J}_4$	$\mathfrak{J}_5$
₳$_1$	(VI, NVC)	(I, M)	(MI, NVC)	(VI, C)	(MI, VC)
₳$_2$	(M, VC)	(MI, C)	(I, C)	(I, VC)	(M, C)
₳$_3$	(I, C)	(VI, VC)	(VI, VC)	(MI, C)	(VI, NVC)
₳$_4$	(M, VC)	(M, VC)	(VI, C)	(M, NC)	(MI, C)
₳$_5$	(I, NVC)	(M, C)	(VI, NC)	(VI, C)	(VI, VC)

Table 4. EXPERT JUDGEMENTS IN THE FORM OF IZNS.

	$\mathfrak{J}_1$	$\mathfrak{J}_2$	$\mathfrak{J}_3$	$\mathfrak{J}_4$	$\mathfrak{J}_5$
₳$_1$	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.25, 0.45, 0.65), (0.15, 0.45, 0.75)⟩)	(⟨(0.75, 0.85, 0.95, 1.0), (0.75, 0.85, 0.95, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95)⟩, ⟨(0.25, 0.45, 0.65), (0.15, 0.45, 0.75)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)
₳$_2$	(⟨(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)	(⟨(0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(0.75, 0.85, 0.95, 1.0), (0.75, 0.85, 0.95, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(0.75, 0.85, 0.95, 1.0), (0.75, 0.85, 0.95, 1.0)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)	(⟨(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)
₳$_3$	(⟨(0.75, 0.85, 0.95, 1.0), (0.75, 0.85, 0.95, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)	(⟨(0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.25, 0.45, 0.65), (0.15, 0.45, 0.75)⟩)
₳$_4$	(⟨(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)	(⟨(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)
₳$_5$	(⟨(0.75, 0.85, 0.95, 1.0), (0.75, 0.85, 0.95, 1.0)⟩, ⟨(0.25, 0.45, 0.65), (0.15, 0.45, 0.75)⟩)	(⟨(0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.45, 0.65, 0.85), (0.35, 0.65, 0.95)⟩)	(⟨(1.0, 1.0, 1.0, 1.0), (1.0, 1.0, 1.0, 1.0)⟩, ⟨(0.65, 0.85, 1.0), (0.65, 0.85, 1.0)⟩)

Step 4: Again, apply to the TIFWA operator and aggregate information of different methodologies as follows:

$$I_{\text{Ⱥ}_1} = \begin{pmatrix}(0.823, 0.428\ 0.147, 0.326),\\(0.323, 0.152, 0.933, 0.654)\end{pmatrix}$$

$$I_{\text{Ⱥ}_2} = \begin{pmatrix}(0.634, 0.43, 0.365, 0.832),\\(0.572, 0.982, 0.352, 0.754)\end{pmatrix}$$

$$I_{\text{Ⱥ}_3} = \begin{pmatrix}(0.654, 0.432\ 0.384, 0.216),\\(0.356, 0.354, 0.864, 0.532)\end{pmatrix}$$

$$I_{\text{Ⱥ}_4} = \begin{pmatrix}(0.634, 0.645, 0.634, 0.431),\\(0.354, 0.462, 0.476, 0.364)\end{pmatrix}$$

$$I_{\text{Ⱥ}_5} = \begin{pmatrix}(0.364, 0.756, 0.985, 0.364),\\(0.657, 0.783, 0.865, 0.835)\end{pmatrix}$$

Table 5. Exerts Judgements in the Form of IZNs.

	$\mathfrak{I}_1$	$\mathfrak{I}_2$	$\mathfrak{I}_3$
Ⱥ_1	$\left(\langle(0.342, 0.042\ 0.125, 0.576),(0.546, 0.352, 0.653, 0.465)\rangle, \langle(0.873, 0.215, 0.215),(0.276, 0.235, 0.109)\rangle\right)$	$\left(\langle(0.532, 0.541\ 0.261, 0.184),(0.984, 0.372, 0.371, 0.462)\rangle, \langle(0.361, 0.165, 0.271),(0.432, 0.362, 0.174)\rangle\right)$	$\left(\langle(0.103, 0.281\ 0.271, 0.193),(0.183, 0.371, 0.754, 0.183)\rangle, \langle(0.193, 0.435, 0.763),(0.193, 0.183, 0.345)\rangle\right)$
Ⱥ_2	$\left(\langle(0.212, 0.012\ 0.113, 0.426),(0.036, 0.332, 0.133, 0.432)\rangle, \langle(0.123, 0.232, 0.315),(0.232, 0.325, 0.121)\rangle\right)$	$\left(\langle(0.213, 0.432\ 0.653, 0.376),(0.736, 0.172, 0.093, 0.022)\rangle, \langle(0.134, 0.254, 0.845),(0.322, 0.745, 0.171)\rangle\right)$	$\left(\langle(0.432, 0.712\ 0.213, 0.916),(0.035, 0.652, 0.533, 0.812)\rangle, \langle(0.543, 0.372, 0.330),(0.082, 0.005, 0.171)\rangle\right)$
Ⱥ_3	$\left(\langle(0.365, 0.032\ 0.215, 0.473),(0.326, 0.542, 0.438, 0.872)\rangle, \langle(0.213, 0.265, 0.435),(0.656, 0.365, 0.276)\rangle\right)$	$\left(\langle(0.092, 0.432\ 0.631, 0.013),(0.636, 0.542, 0.332, 0.212)\rangle, \langle(0.245, 0.325, 0.905),(0.096, 0.215, 0.254)\rangle\right)$	$\left(\langle(0.721, 0.621\ 0.463, 0.173),(0.341, 0.042, 0.401, 0.212)\rangle, \langle(0.209, 0.015, 0.015),(0.326, 0.915, 0.541)\rangle\right)$
Ⱥ_4	$\left(\langle(0.342, 0.643\ 0.755, 0.637),(0.566, 0.364, 0.673, 0.735)\rangle, \langle(0.764, 0.376, 0.235),(0.273, 0.385, 0.989)\rangle\right)$	$\left(\langle(0.342, 0.643\ 0.755, 0.637),(0.566, 0.364, 0.673, 0.735)\rangle, \langle(0.764, 0.376, 0.235),(0.273, 0.385, 0.989)\rangle\right)$	$\left(\langle(0.091, 0.463\ 0.155, 0.427),(0.426, 0.184, 0.833, 0.005)\rangle, \langle(0.164, 0.431, 0.643),(0.032, 0.193, 0.013)\rangle\right)$
Ⱥ_5	$\left(\langle(0.347, 0.445\ 0.326, 0.478),(0.099, 0.372, 0.481, 0.487)\rangle, \langle(0.373, 0.435, 0.985),(0.216, 0.435, 0.643)\rangle\right)$	$\left(\langle(0.218, 0.736, 0.432, 0.018),(0.319, 0.652, 0.011, 0.917)\rangle, \langle(0.323, 0.015, 0.173),(0.016, 0.035, 0.531)\rangle\right)$	$\left(\langle(0.831, 0.4326, 0.016, 0.403),(0.731, 0.322, 0.511, 0.621)\rangle, \langle(0.821, 0.015, 0.011),(0.912, 0.035, 0.193)\rangle\right)$

	$\mathfrak{I}_4$	$\mathfrak{I}_5$
Ⱥ_1	$\left(\langle(0.461, 0.381\ 0.365, 0.826),(0.326, 0.432, 0.213, 0.655)\rangle, \langle(0.213, 0.232, 0.475),(0.237, 0.335, 0.379)\rangle\right)$	$\left(\langle(0.211, 0.211\ 0.391, 0.021),(0.832, 0.061, 0.091, 0.135)\rangle, \langle(0.318, 0.012, 0.015),(0.293, 0.105, 0.341)\rangle\right)$
Ⱥ_2	$\left(\langle(0.421, 0.742\ 0.215, 0.216),(0.521, 0.322, 0.133, 0.215)\rangle, \langle(0.343, 0.235, 0.455),(0.326, 0.165, 0.121)\rangle\right)$	$\left(\langle(0.523, 0.432\ 0.282, 0.643),(0.384, 0.032, 0.741, 0.632)\rangle, \langle(0.023, 0.625, 0.625),(0.376, 0.185, 0.421)\rangle\right)$
Ⱥ_3	$\left(\langle(0.212, 0.342\ 0.215, 0.566),(0.216, 0.752, 0.653, 0.465)\rangle, \langle(0.323, 0.455, 0.675),(0.126, 0.565, 0.124)\rangle\right)$	$\left(\langle(0.371, 0.632\ 0.375, 0.214),(0.308, 0.391, 0.923, 0.005)\rangle, \langle(0.227, 0.625, 0.215),(0.213, 0.005, 0.434)\rangle\right)$

$Ⱥ_4$	$\begin{pmatrix} \langle (0.334, 0.212\ 0.365, 0.676), (0.543, 0.322, 0.213, 0.754) \rangle \\ \langle (0.133, 0.565, 0.425), (0.386, 0.935, 0.279) \rangle \end{pmatrix}$	$\begin{pmatrix} \langle (0.214, 0.562\ 0.815, 0.216), (0.213, 0.462, 0.183, 0.184) \rangle \\ \langle (0.210, 0.960, 0.295), (0.319, 0.425, 0.202) \rangle \end{pmatrix}$
$Ⱥ_5$	$\begin{pmatrix} \langle (0.012, 0.345\ 0.835, 0.166), (0.316, 0.542, 0.153, 0.175) \rangle \\ \langle (0.323, 0.185, 0.325), (0.164, 0.655, 0.219) \rangle \end{pmatrix}$	$\begin{pmatrix} \langle (0.721, 0.655\ 0.725, 0.310), (0.028, 0.312, 0.523, 0.285) \rangle \\ \langle (0.281, 0.614, 0.281), (0.015, 0.715, 0.141) \rangle \end{pmatrix}$

Step 4: Compute the value of the score function Using Eq. 10.

$$\mathcal{M}(I_{Ⱥ_1}) = 0.4756, \mathcal{M}(I_{Ⱥ_2}) = 0.8566, \mathcal{M}(I_{Ⱥ_3}) = 0.7836,$$

$$\mathcal{M}(I_{Ⱥ_4}) = 0.4786, \mathcal{M}(I_{Ⱥ_5}) = 0.6572$$

Step 5: Ranking of different score functions are extracted as follows:

$$Ⱥ_2 \succ Ⱥ_3 \succ Ⱥ_5 \succ Ⱥ_4 \succ Ⱥ_1$$

Figure 7 also illustrates the ranking of alternatives based on computed score functions. From this figure, we can examine the $Ⱥ_2$ is a more dominant optimal option.

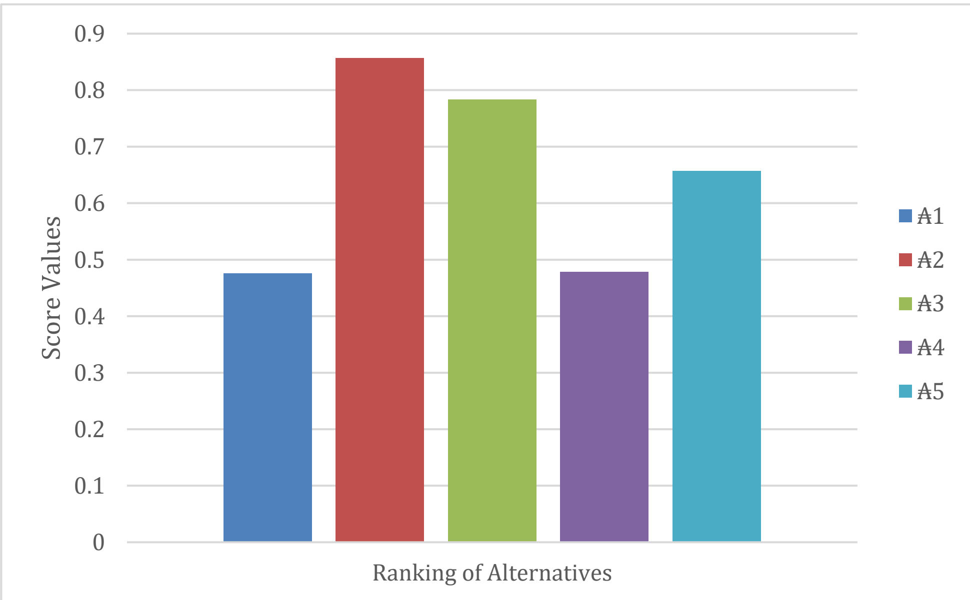

Fig 7. Ranking results of discussed educational strategies based on computed results of score functions.

5. Comparative Study

This section is articulated to conduct a comprehensive comparison technique for contrasting the results of pioneered aggregation operators with previous approaches developed by different mathematicians and research scholars. To serve this purpose, we employed the mathematical approaches of different mathematicians. For instance, Nik Badrul Alam *et al.*, [47] proposed a decision-making technique to aggregate the IFZNs. Liu *et al.*, [48] anticipated an intelligent decision-making model to investigate the ranking of alternatives based on particular criteria or attribute information. Haktanır and Kahraman [44] estimated the ranking of optimal options using the theory of AHP and the TOPSIS method under consideration of IFZNs. Nik Badrul Alam *et al.*, [49] discussed an optimization technique and arithmetic aggregation operator to deduce the ranking of alternatives or preferences. Furthermore, we also applied some novel approaches to intuitionistic fuzzy information and decision-making models. Hussain *et al.*, [50] characterized new aggregation operators of Hamy mean aggregation operators with a decision analysis model. Ullah *et al.*, [51] utilized the robust operation of Sugeno-Weber t-norms to deduce mathematical approaches and decision-making techniques. After applying previous mathematical approaches and decision analysis models, we obtained a ranking of alternatives based on aggregated results by the existing terminologies. Table 6 demonstrates ranking results estimated by using computed score functions.

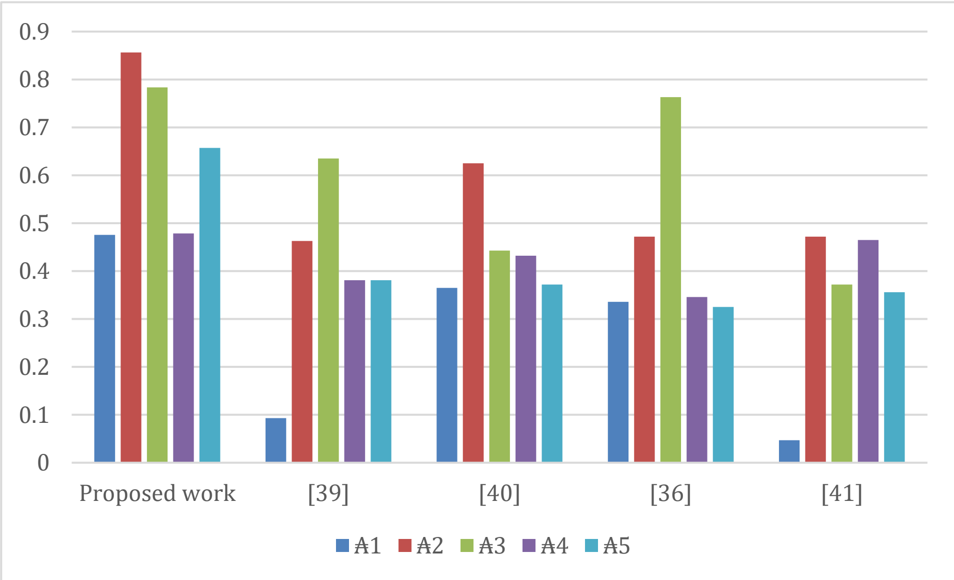

Fig 8. Illustrate the ranking results of the comparative study.

From Table 6, we can examine a few mathematical approaches [50], [51] and decision-making models are unable to handle the given information in Table 3. After carefully examining the ranking results by comparative studies, we concluded that our diagnosed terminologies are more superior and effective mathematical approaches. Additionally, Figure 8 also illustrates computed results of score functions by existing approaches.

Table 4. Expert Judgements in the Form of IZNs.

Environments	Rankin of preferences
Proposed work	$₳_2 \succ ₳_3 \succ ₳_5 \succ ₳_4 \succ ₳_1$
Nik Badrul Alam *et al.*, [47]	$₳_3 \succ ₳_2 \succ ₳_5 \succ ₳_4 \succ ₳_1$
Liu *et al.*, [48]	$₳_2 \succ ₳_3 \succ ₳_4 \succ ₳_5 \succ ₳_1$
Haktanır and Kahraman [44]	$₳_3 \succ ₳_2 \succ ₳_5 \succ ₳_1 \succ ₳_5$
Nik Badrul Alam *et al.*, [49]	$₳_2 \succ ₳_4 \succ ₳_3 \succ ₳_5 \succ ₳_1$
Hussain *et al.*, [50]	Limited structure
Ullah *et al.*, [51]	Limited structure

6. Conclusion

This article articulated a robust ranking technique based on the MADM problem. However, each decision-making technique has several advantages and disadvantages. This manuscript expanded the theory of Z-number to deduce IZNs. The IZNs are more feasible and efficient approaches used to handle human opinion more accurately. The IZNs are the combination of restriction and reliability components of PMG and NMGs. In this article, we applied the mathematical approaches of the TIFWA operator to resolve complicated real-life applications. By applying derived mathematical approaches, we deduce the ranking of alternatives based on the computed score function by aggregating IZNs. An intelligent decision-making model of the MADM problem is used to execute a ranking of suitable ideological and political education strategies under consideration of dominant resources and their impact on talent management. After evaluating judgments of expert opinion, we examine the ranking of alternatives based on the score functions of different optimal options. From the findings of the aggregation process, we examined $Ꞩ_2$ is more appropriate option from different considered strategies. The comparative study is established to reveal the supremacy and superiority of proposed theories with existing mathematical terminologies of IZNs.

In the coming future, we can utilize IFZNs for handling different uncertain information using advanced decision-making methodologies of Parsimonious Best Worst Method [52], LMAW and DNMA methods [53].

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